Equilibrium Real Interest Rate in Brazil: Convergence at Last*

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Abstract

Real interest rates in Brazil, at least the short term, has converged to zero in in 2020. The main purpose of this paper is to measure the equilibrium interest rate to assess the stance of the monetary policy. We calculated this latent variable using different methodologies, including a version of Laubach and Williams (2003) with fiscal and credit variables. Based on this approach, the long run equilibrium rate is in the range of 2-3% depending on the output gap and risk scenario. Our sensitivity analysis has shown that our results changed slightly for different scenarios for Brazil risk premium but deeply regarding potential GDP growth. We also notice that since 2019 the effective real rate is significantly below the neutral rate and slightly below the Taylor rate, which means an expansionary monetary policy lately. The real interest rate based on the this estimated Taylor rule should be at -0.8% in the 20Q3

Keywords: real interest rate, Laubach Willians, Kalman Filter.

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1- Introduction

After controlling inflation with the launch of the Real plan in 1994, Brazil has finally been able to converge the real interest rate to a new level close to zero at the end of 2019s, just before the pandemic.

Nowadays, when we are at the end of the easing cycle, central bank policy rate (Selic rate) is at record low level at 2%. The effective real interest rate (360-PreDI Swap discount by 12-month inflation expectation) reached -0.75% in June 2020 the lowest level ever, and it was at the end of August slightly below zero as one can see at Table 1. Given there is still idle capacity in the economy, it is possible that the effective rate is below the equilibrium rate. Hence, two questions that naturally follows: (i) what is the equilibrium rate? (ii) is it the monetary policy accommodative indeed?

Table 1 - Selic and Real Rate

In this paper we will define the equilibrium interest rate as the one that stabilize the economy driving the inflation to the target and the output gap to this potential level.

Magud e Tsounta (2012) summarizes the methodologies to calculate the equilibrium interest rate in two big groups:

- Static Methodologies:
    - Consuming Smoothing Models;
    - Uncovered Interest Parity;
- Dynamic Methodologies:
    - Hodrick-Prescott (HP) Filter;

- Implicit Common Stochastic Trend;
- Dynamic Taylor Rule;
- Taylor Rule with Augmented Inflation Expectation;
- General Equilibrium Model.

This is the third paper in which we aimed to measure the equilibrium real interest rate in Brazil with different approaches as Miranda and Muinhos (2002) and Muinhos & Nakane (2006). Previous we performed direct measures from IS curve, panel with different emerging countries, information on the yield curve, even trying to extract the equilibrium rate from marginal productivity. However, using state space in similar fashion as done by Laubach and Williams (2003) was not performed.

Laubach and Williams (2003) focus their work in estimating the real interest rate – the real interest rate consistent with output equalizing potential and stable inflation – on a medium-run concept of price stability that not considers the effects of short-run price and output fluctuations. Their purpose is to show that the time variation in natural interest rate is important to the analyses and the performance of monetary policy and its real-time mismeasurement can cause a significant deterioration in macroeconomic stabilization.

Based on the definition of the natural rate of interest considering deviation of output from potential, the natural rate of interest estimation also entails finding the potential output as well. Moreover, giving the linkage between natural interest rate and the trend growth rate, they have to estimate both the level of potential output and its trend growth rate. Therefore, they use Kalman filter to estimate these unobserved variables the potential output trend growth rate. Besides Kalman filter, they model the cyclical dynamics of output and inflation using a restricted VAR model and then, using median-unbiased estimates of these coefficients, based on Stock and Watson (1998), they apply maximum likelihood to estimate the remaining model parameters.

After estimating the model using quarterly U.S. data over the period 1961 to 2000, Laubach and Williams realize an exercise in which they use simulations of the estimated model to assess the effects of natural interest rate mismeasurement. In addition, they found that mismeasurement leads to a significant deterioration in output stabilization but has relatively modest effects on inflation stabilization.

Barcelos Netto and Portugal (2008) presented the first attempt to calculate the natural interest rates using the Laubach and Williams methodology for Brazil. However, given that the period of estimation ended in 2005, in the first stages of inflation targeting in Brazil, the outcome of the estimation shows a rate hovering 10%, which is significantly greater the what we expect to the range nowadays.

Araujo and Silva (2012) also present some different methodologies of measuring the Brazilian neutral real interest rate: i) statistical filters; ii) a state space macroeconomic model. They include variables such as the real exchange rate, credit default swap and an international interest rate. In the period that they considered, from 2002 up to the end of 2012, they found the country´s natural rate of interest to be around 3.5%.

Perelli Roache (2014) also followed the same approach trying to measure the equilibrium interest rate using statistical filters, short and long run estimation of IS curve micro-founded models and even state space model similar to Goldfajn and Bicalho (2011), but any of the adopted methodologies are not even close to Laubach and Williams (2003).

The objective of this paper is to measure the equilibrium interest rate in Brazil using different methodologies.

In the first approach, based on Schulz (2019), we combined statics and dynamic approaches, starting with simple ones like the long run real interest rate average, ending up in a dynamic Taylor Rule within fixed-effect panel with 27 emerging countries quarterly from 1995-2019. In this approach, we also included a simple Taylor rule and a Hodrick-Prescott filter.

The second approach is simplified version of the Laubach Williams (2003) approach for Brazil, when we considered only the equilibrium interest rate as a state variable and the output gap as a exogenous one. , We also included fiscal and credit variables as explanatory variables in the process. We also add a risk premium variable in the equilibrium interest rate equation and we present a new methodology to calculate the output gap.

The third approach is a updated of the papers Goldfajn and Bicalho (2011), Perreli and Roache (2014) and Augusto (2018), extending the period from 2003 up to 2020 to take a better view of the variables that allowed the recent real interest rate conversion to low levels.

The following section presents the Taylor rule methodology and the new variables that we included in the model. In the third session, we present our version of the Laubach Williams (2003). In the fourth, we show the update version on the long and short interest rate approach. In the fifth section we compared the effective interest rate with a average of the semi-Laubach Williams and an estimated Taylor rule. In the last section, we summarized and concluded the paper.

2- Taylor Rule approach

In this section of the paper, the neutral Interest Rate (NIR) is estimated by four models: long-term average of the real interest rate, Hodrick and Prescott (HP) filter, a standard Taylor Rule and Dynamic Taylor Rule - the latter, through a panel data regression with fixed effects, for the period 1995-2019 in quarterly terms. The results show that: (a) Brazilian interest rate is high in its neutral, nominal and real terms (compared to other emerging economies); (b) NIR has declined consistently over the past few decades.

2.1 Methodology

a. Long Run Real Interest Rate (RIR)

Based on the model of Miranda and Muinhos (2003), NIR can be estimated as a long-term trend. In this case, estimated by the arithmetic mean of the RIR over the five-year period (20 quarters), with the final long-term RIR estimate being given by the average of the estimates for these four periods

b. HP Filter

NIR is estimated using the Hodrick Prescott filter using quarterly RIR data between the period 1985Q1-2025Q4, using projections when available from the IMF or OECD, when not, stretching the value of the last period, so to reduce possible distortions at the extremes (periods tending to 1995 and 2019) by the filter.

c. Standard Taylor Rule

The Taylor rule is a standard monetary policy response in which the monetary authority reacts from an inflation deviation to the target and from a output deviation to the potential.

A Taylor rule generalized version is given by:

$$i_t - i^* = a_\pi(\pi_t - \pi^T) + b_y(y_t - y^*) \qquad (1)$$

where:

- i_t: nominal interest rate at time t;
- i^*: nominal neutral rate;
- π_t: inflation at period t;
- π^T: inflation targeting
- y_t: output (%GDP) in periold t;
- y^*: potential output (%GDP);

- a_{π}: interest elasticity to inflation;
- b_y: interest elasticity to output.

In this approach, we estimate NIR (r^*) from a Taylor rule subtracting i^* in Equation ¨by Equation 1 e isolating r^* which is following this procedure a function of: neutral nominal interest rate ($i_{t,p}$), inflation ($\pi_{t,p}$ *and* $\pi^T_{t,p}$), output ($y_{t,p}$ and $y^*_{t.p}$).

$$r^*_{t,p} = i_{t,p} - \pi^T_{t,p} + a_{\pi}\left(\pi^T_{t.p} - \pi_{t,p}\right) + 0{,}5\left(y^*_{t.p} - y_{t,p}\right) \qquad (2)$$

Instead of estimating a_{π}, it was considered two calibrated values. In the first case (TYL1), $a_{\pi} = 0.5$ that is the value suggested by Taylor (1993) to the US economy. In the second case (TYL2), we considered $a_{\pi} = 1.0$ which shows a stronger commitment against inflation.

Regarding output gap $\left(y^*_{t,p} - y_{t,p}\right)$, it was used the IPEA series.

For inflation expectation used the medium-long run inflation based on the 2 year ahead average to ease international comparison according to the equation ($\pi^T_{t,p}$):

$$\pi^T_{t,p} = \frac{\sum_t^{t+8} \pi_{t,p}}{8} \qquad (3)$$

d. Dynamic Taylor Rule in Panel Data

Judd and Rudebusch (1998) propose a model in which the central bank can dynamically adjust the interest rate according to the dynamics of macroeconomic variables in period t. Equation 16 was re-specified as:

$$i^*_t = \pi_t + r^* + \lambda_1(\pi_t - \pi^T) + \lambda_2 gap_t + \lambda_3 gap_{t-1} \qquad (4)$$

In this case, i^*_t is the recommended interest rate in order to central bank to adjust the economy gradually. *gap* is the output gap $(y - y^*)$ which enters not only contemporaneously but also with a lag.

The dynamics of the adjustment in which the current level of the nominal interest rate is related - obtained through some indicator of the interest rate observed in the market (in the original article, the federal interest rates of the United States are used) - with the recommended interest rate is given by equation 6:

$$\Delta i_t = \gamma(i^*_t - i_{t-1}) + \rho \Delta i_{t-1} \qquad (5)$$

Where, γ is a sensitivity coefficient of the nominal interest rate first difference to the nominal interest rate gap and ρ is the sensitivity coefficient of the nominal interest rate first difference to the lagged nominal interest rate first difference with one lag.

Substituting equation (5) in (4) and isolating the first difference from the nominal interest rate on the left side, the following equation is obtained:

$$\Delta i_t = \gamma\alpha - \gamma i_{t-1} + \gamma(1+\lambda_1)\pi_t + \gamma\lambda_2 gap_t + \gamma\lambda_3 gap_{t-1} + \rho\Delta i_{t-1} \quad (6)$$

In this model α is the constant that contains the neutral interest rate, $\alpha = r^* - \lambda_1\pi^T$. According to Judd & Rudebusch (1998), r^*e π^T are inseparably combined in the constant α when estimating the model. Leonardi (2003) obtained a variation to estimate the model in a panel for each country p:

$$\Delta i_{p,t} = \alpha - \beta_1 i_{p,t-1} + \beta_2\Delta i_{p,t-1} + \beta_3\pi_{p,t} + \beta_3 gap_{p,t} + \beta_3 gap_{p,t-1} + \varepsilon \quad (7)$$

We estimated equation 7 using real interest rate (r) instead of (i). We also used fixed effects in the panel data model given that each country may have particularities that are invariant over time and that are not related to the other regressors - that is, specific characteristics of the country p itself . In Schulz (2019), one can see this panel for 26 countries.

In Table 1, it is presented the neutral rates estimations only for Brazil and for the emerging country average, based on the methodologies described above. It is clear a downward trend since 1999 in all procedures for Brazil. However, in all the procedures, in the last period 2015-2019, interest rate in Brazil is greater than the world average, but TY2.

The last interest rate estimations based on Taylor rules are negative (-0.16% TYL1 and -0.78% TYL2). The HP has declined up to 1.12%, but the panel result can be only extracted for the entire period 2015-2019 stands at 3.3% as shown in Table 1.

Table 1 Dynamic Taylor Rule Panel

Country	Model	Neutral Interest Rate (r*) for each period (% a.a.)					
		1995-1999	2000-2004	2005-2009	2010-2014	2015-2019	Média
Average	RIR	0.0	2.7	0.5	0.1	0.9	0.8
	MD LP	1.3	0.9	0.8	0.9	1.1	1.0
	F.HP	2.2	2.1	0.5	0.2	0.8	1.2
	TYL1	-0.3	2.2	0.1	0.3	0.7	0.6
	TYL2	-4.0	1.9	-0.1	0.1	0.7	-0.3
	PNL	23.1	3.1	2.2	2.9	1.2	6.5
Brasil	RIR	7.0	9.5	8.2	4.0	4.1	6.5
	MD LP	16.0	9.7	5.7	4.3	3.4	7.8
	F.HP	15.8	11.7	7.6	4.2	3.8	8.6
	TYL1	-12.3	8.2	6.9	4.9	1.5	1.8
	TYL2	-33.4	8.2	6.6	5.6	0.3	-2.6
	PNL	25.9	6.3	5.6	6.7	3.3	9.6

RIR: Real Interest Rate. MD LP: Long Run Average. F.HP: HP filter. TYL1: Taylor Rule 1. TYL2: Taylor Rule 2. PNL: Panel with Dynamic Taylor RuleModelo.
Source: Schulz (2019).

3- The Semi-Laubach-Williams Approach

We based the approach on Laubach Williams (2003), in which we add some special features to include some characteristics of the Brazilian economy. Following Laubach Williams (2003) and Araujo e Silva (2013), the output gap fluctuations are attributed to real interest gap to a central tendency, which is the real equilibrium rate. In fact, it is not the real interest rate that matters but the difference between the effective real rate and the equilibrium one. It is an augment version of the IS curve in which the dependent variable is the output that depends on the real interest rate gap, on the credit conditions and on central government expenditures.

$$h_t = \beta_1 h_{t-1} + \beta_2 [r_t - \left(sv_t + \beta_3 \Delta GDP^*_{4t} + r^{US}_t + CDS^{5y}_t\right) + \beta_4 FCI_{t-1} + \beta_5 \Delta g_t + \beta_6 X_t + \beta_7 D^{08}_t + \varepsilon_t \quad (8)$$

$$sv_t = sv_{t-1} + \vartheta_t \quad (9)$$

The term inside the brackets is a representation of an interest gap. The neutral rate is the part on the parentheses as shown below.

$$r^*_t = \left(sv_t + \beta_3 \Delta GDP^*_{4t} + r^{US}_t + CDS^{5y}_t\right) \quad (10)$$

The first term of equation 3 is the state variable of the system following a very simple ar(1) structure estimated by the Kalman filter. This approach recursively calculates non-observable components using past data. The other terms are the structural part of the equation. The original paper has only the average of potential product growth as a structural variable. For this paper, we include the US interest rate and the Brazilian risk premium measured by the 5-year sovereign credit default swaps (CDS).

$$r^*_t = \left(sv_t + \beta_3 \Delta GDP^*_{4t} + r^{US}_t + CDS^{5y}_t\right) \quad (11)$$

3.1- The Data

Below we explain how we obtained and treated the variables used in our estimations.

Real Rate (r)– it is the Selic rate deflated by 12-month ahead inflation expectation.

ΔGDP^*_{4t} 4-quarter increasing in our default potential output growth

r^{US}_t3-month US Treasury rates

CDS_t^{5y} Brazilian risk premium measured by Credit Defaut Swaps (with 5 year mature). The variable used in the estimation is the residual of the risk premium against the output gap to avoid endogeneity.

FCI financial condition index. This variable is year over year household credit growth controlled by output gap and Selic rate as well**.**

Δg_t is the first difference in central government expenditures measured in BRL terms.

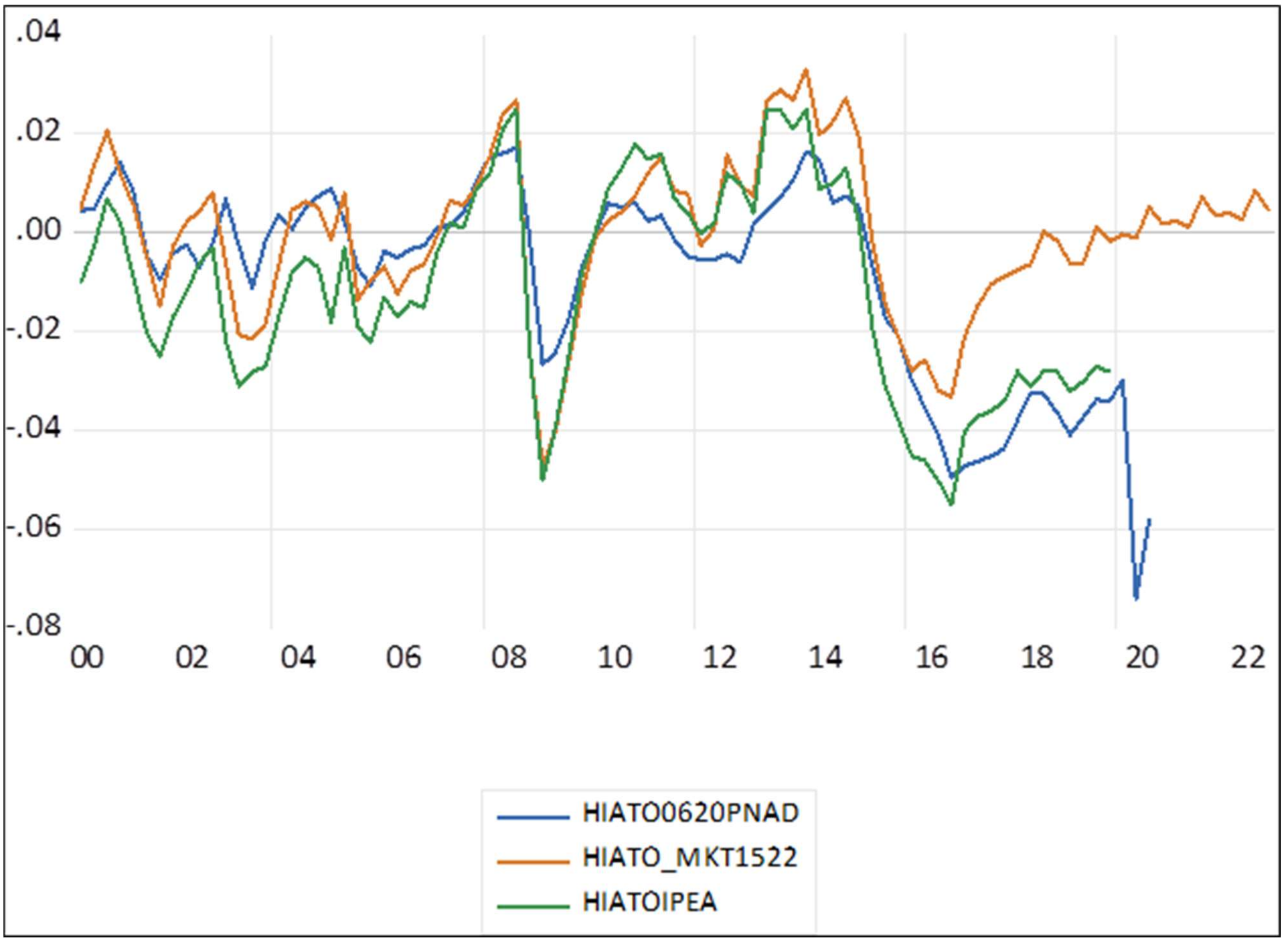
Output gap (h)**:** our standard measure is calculated as a Hodrick-Prescott filter with a special feature given that the end of the sample period is not the last quarter with data available**.** We extend our sample up to 2022 using GDP growth Focus consensus forecast. The reason for that is to avoid end-point bias in Hodrick-Prescott estimation.

As an alternative procedure, we use an output gap, which is a weighting average between labor market and industrial capacity utilization slackness as describes in Muinhos and Alves (2003).

Even controlling for the end-point, one can see that the default output gap has a leading recovery comparing to the alternative measure. Both series have a minimum point at -5% at the end of 2016. However, the alternative GDP measure has not recovered significantly in 2017 still presenting an average in comparison to 3% on the default output gap, showing perhaps a premature recovery.

We also included one extra alternative output series HiatoIPEA with is a calculated by IPEA using a proper series for potential output based on Cob-Douglas production function.

Figure 2 - Different Output Gaps



3.2 – Empirical Results

3.2.1 Estimation Results

We ran 10 different version of our augmented IS curve. The first one is the closest version to Laubach and Williams (2003). The two extension to the IS curve (credit and central government expenditures) are significant and with the expected sign in all specification as one can see in Table 2. Central government expenditures present the correct sign in all specification, whereas credit is significant at 10% in 4 models and highly significant in Model 17. However, regarding the terms that form the real equilibrium rate (r*) the coefficients that are significant in all of specification. The average of potential output growth is significant in 4 out of 10 regressions. The US interest rates are not significant in any of the specification and the Brazilian risk premium has the correct sign and is statistically significant in the both equation (9 and 19). The state space variable sv1(equation 2) is significant in most of the estimations with coefficient value slightly below the end-point equilibrium real interest rate calculated by equation 3.

Table 2- IS Estimation Results

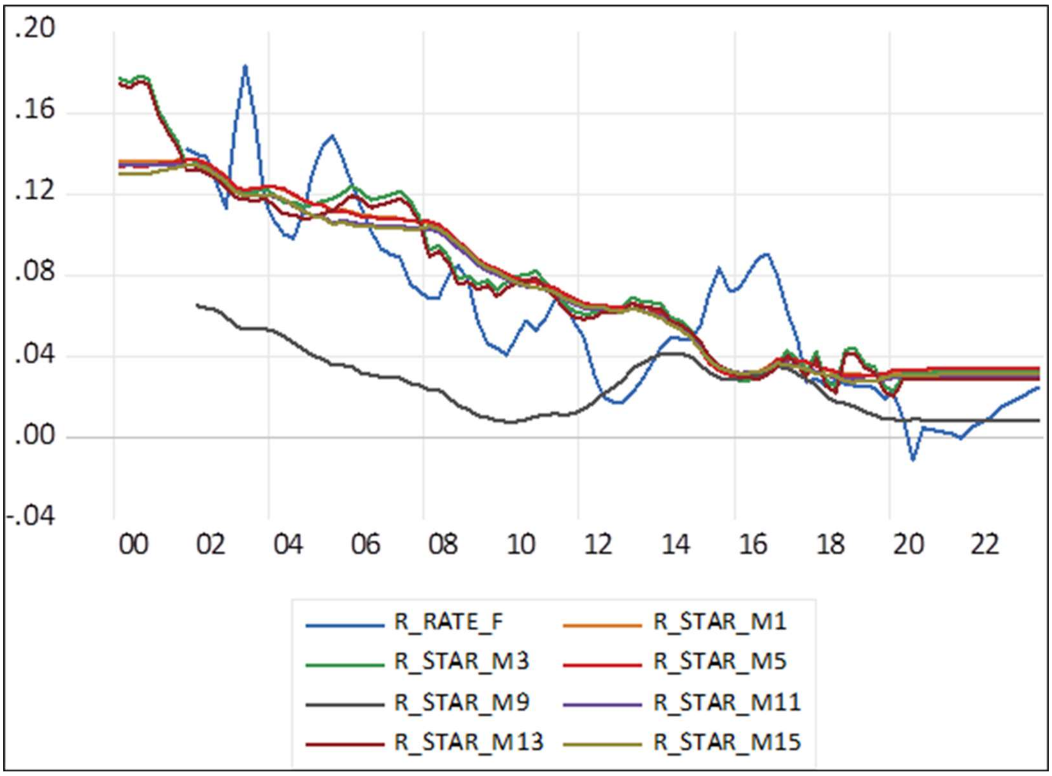
	Output -1	R-rate	Potential GDP	Credit	CG Expend	Dum Crises	US inter	cds	sv1
	b(1)	b(2)	b(3)	b(4)	b(5)	b(6)	b(7)	b(8)	
Model 1	**0.74**	**-0.14**	0.73		**0.1**	**-0.04**			**0.028**
	0.03	0.02	0.46		0.014	0.002	0		0.013
Model 3	**0.74**	**-0.132**	0.33		**0.1**	**-0.039**	-1		**0.037**
	0.03	0.021	0.46		0.013	0.0027			0.014
Model 5	**0.75**	**-0.137**	0.93		**0.11**	**-0.042**	-0.25		**0.026**
	0.034	0.025	0.53		0.014	0.0038	0.49		0.014
Model 7	**0.74**	**-0.12**	**1.21**		**0.1**	**-0.041**	-1	-1	**0.04**
	0.034	0.018	0.45		0.014	0.0028			0.015
Modelo 9	**0.74**	**-0.14**	**1.19**		**0.1**	**-0.042**	-0.075	**0.52**	**0.031**
	0.035	0.026	0.53		0.015	0.002	0.49	0.24	0.014
Model 11	**0.76**	**-0.13**	0.63	0.03	**0.1**	**-0.04**			**0.027**
	0.03	0.019	0.51	0.018	0.014	0.003			0.013
Model 13	**0.76**	**-0.12**	0.24	0.03	**0.1**	**-0.039**	-1		**0.035**
	0.035	0.021	0.52	0.018	0.014	0.003			0.015
Model 15	**0.77**	**-0.13**	0.86	0.03	**0.11**	**-0.041**	-0.29		0.024
	0.03	0.024	0.57	0.019	0.014	0.003	0.55		0.014
Model 17	**0.76**	**-0.11**	**1.04**	**0.035**	**0.1**	**-0.04**	-1	-1	**0.039**
	0.034	0.017	0.52	0.017	0.015	0.003			0.015
Model 19	**0.76**	**-0.127**	**1.13**	0.031	**0.1**	**-0.041**	-0.1	**0.55**	**0.028**
	0.035	0.025	0.57	0.018	0.015	0.003	0.55	0.26	0.014

Source: Centro Macro-Brasil:

3.2.2 - Sensitivity Analysis

As one can see in Figure 3, our simulations of the real equilibrium rate converges to an average of 2.9% in the last quarter of 2023. The graphical representation is distributed in a close range from 0.9% in Model 1 up to 3.4% in Model 5, considering HP filter output gap (Hiato1515) and normal CDS case. It is worth noticing that 20Q1 just before the pandemic crises was 2.5%. Hence, according to our estimation Brazil is running nowadays an expansionary monetary policy, but only about 200 bps below neutral.

Figure 3 - The Equilibrium Real Interest Rates



The terminal conditions matter regarding the variables that we consider exogenous in our simulation. Hence, in the situation that we called normal condition, we considered CDS at 170 bps and GDP growth at 1.5%.

Table 3 Equilibrium Interest Rates Scenarios

	Hiato0620		HiatoIPEA		Hiato1515	
	Average		Average		Average	
	2020Q2	2023Q4	2020Q2	2023Q4	2020Q2	2023Q4
High CDS	0.05	3.1	3	4.1	2.9	4.2
Normal	0.05	1.7	3	2.7	2.8	2.9
Low CDS	0.05	1.4	3	2.5	2.9	2.7

Based on that, we consider some sensitivity analysis in our simulation. In the case of a worsening of the international condition or internal fiscal position, our hypotheses are that CDS moves gradually to 300 in the end of the horizon. In this case, the real equilibrium rate will reach 3.1% (Hiato062) in average and 4.2% in Hiato1515. On the other hand, in case of CDS getting lower reaching 100 in 2021; real equilibrium rate also decreases to 1.4% in average and 2.7% when we run the HP filter output gap (Hiato1515).

The sensitivity analysis for GDP growth is more puzzling and depends a lot in what measured of output gap is used. Considering another measure of output gap (Hiato0620), the equilibrium interest rates are significantly smaller. Our alternative GDP measure has a negative level of less than 1% in average in 20Q1 in comparison to 2.8% on the default output gap. The equilibrium real rate is 1.7% when we used this alternative output gap.

4 – The Long and Short Run Approach

The third methodology is an updated of the papers Goldfajn and Bicalho (2011), Perreli and Roache (2014) and Augusto (2018), in which we extended the period from 2003 up to 2020, using the long-run and short-run equations.

According to Bernhardsen and Karsten (2007), the long-run equilibrium rate is determined by economic fundamentals as potential GDP, saving decision, and temporary supply and demand shocks.

Bernhardsen and Karsten (2007) claim that there are deviations in the short run from the long run equilibrium rate, but the short run should float around the long run one due to temporary shocks. They used the Taylor rule to figure out the short run rate considering the output and inflation gap going to zero.

In the search for the short-term equilibrium interest rate, Goldfajn and Bicalho (2011) claim that this rate might be affect by short run variables moving apart temporarily from the long-term rate that is affect by the fundamentals.

4.1 Methodology

The long-term equation is based on the following equation:

$$r_t = \beta_0 + \beta_1 t + \beta_2 X_t + \varepsilon_t \quad (12)$$

r_t is the observed real interest rate

β_0 is the constant

β_1 is the coefficient on the linear trend

t is the linear trend

β_2 is the coefficient of the exogenous variables

X_t is the vector of exogenous variables that affect the long term interest rate

ε_t are the transitory shocks.

According to G&B the short-term equilibrium interest rate appears from broader IS curve specification.

$y_t - \bar{y}_t = \alpha_0(y_{t-1} - \bar{y}_{t-1}) + \propto_1 (y_t^m - \bar{y}_t^m) + \alpha_2(r_t - \bar{r}_t) \propto_3 (G_t - \bar{G}_t) + \propto_4 \left(C_t^d - \bar{C}_t^d\right) + \propto_5 (e_t - \bar{e}_t) + \varepsilon_t$ (y_t is the Brazilian GDP

$\bar{y}_t$ is the Potential Brazilian GDP

y_t^m is the US GDP

$\bar{y}_t^m$ is the Potential US GDP

G_t is the government expenditures

$\bar{G}_t$ is the filtered government expenditures

C_t^d is the ear-marked credit

$\bar{C}_t^d$ is the filtered ear-marked credit

e_t is the real exchange rate

$\bar{e}_t$ is a smoothed exchange rate

ε_t are transitory shocks.

In order to estimate the IS curve, it was necessary to use the first difference of the output gap:

$$(y_t - \bar{y}_t) - (y_{t-1} - \bar{y}_{t-1}) = \alpha_0\big((y_{t-1} - \bar{y}_{t-1}) - (y_{t-2} - \bar{y}_{t-2})\big) + \propto_1 (y_t^m - \bar{y}_t^m) + \alpha_2(r_{t-2} - \bar{r}_{t-2}) + \propto_3 (G_{t-2} - \bar{G}_{t-2}) + \propto_4 \left(C_{t-1}^d - \bar{C}_{t-1}^d\right) + \propto_5 (e_{t-4} - \bar{e}_{t-4}) + \varepsilon_t \quad (14)$$

In order to find the short-term interest rate the output gap is set to be zero

$$y = \bar{y} \quad (15)$$

So from equation 10, we can find the short term interest rate as:

$$r_t = \bar{r} - \frac{1}{\propto_2}\left[\propto_1 (y_t^m - \bar{y}_t^m) + \propto_3 (G_{t-2} - \bar{G}_{t-2}) + \propto_4 \left(C_{t-1}^d - \bar{C}_{t-1}^d\right) + \propto_5 (e_{t-4} - \bar{e}_{t-4})\right] \quad (16)$$

4.2 Empirical Results.

As one can see in Table 4, the real interest rate lag variables are highly significant, and the sum of the coefficients are 86%, showing high inertia.

Likewise, the forementioned papers the Brazilian risk measured by CDS 5 year is high significant and positive, which means that an worsening in risk perception increases long run interest rate.

In comparison to Augusto (2018), government debt and industrial confidence are significant, but government debt presents an unexpected coefficient sign. Inflationary surprise in line with Perreli and Roache and Augusto (2018) is significant and presents the expected sign.

Table 4 Long Run Interest Rate

Variable		Long-Term
C		0,25
		(0,0325)***
Real Interest Rate t-1		0,97
		(0,095)***
Real Interest Rate t-2		-0,26
		(0,082)**
CDS 5Y		0,038
		(0,0014)**
Credit t-5		-0,2528
		(0,004)***
Gross Debt t-2		-0,434
		(0,0073)***
Inflation Surprise t-3		0,0012
		(0,0004)***
Ind Confidence t-2		0,0266
		(0,01)**
Dummy 09Q1		-0,023
		(0,0021)***
Dummy 11Q1 2 13Q2		-0,010
		((0,0021)***
R2 - ajusted		0,975
F Statistic		293,38

Table 5 shows the results for the IS curve (equation 14) estimated in first difference. The results are similar the ones found by Augusto (2018). The only significant coefficients are the US GDP, the ear marked credit, and the long run interest rate gap.

The negative sign of ear-marked credit might show that this variable is not exogenous regarding output gap, meaning that whenever the economy was weakening, there was an incentive to federal banks to boost credit.

Table 5 IS Curve in 1st Difference

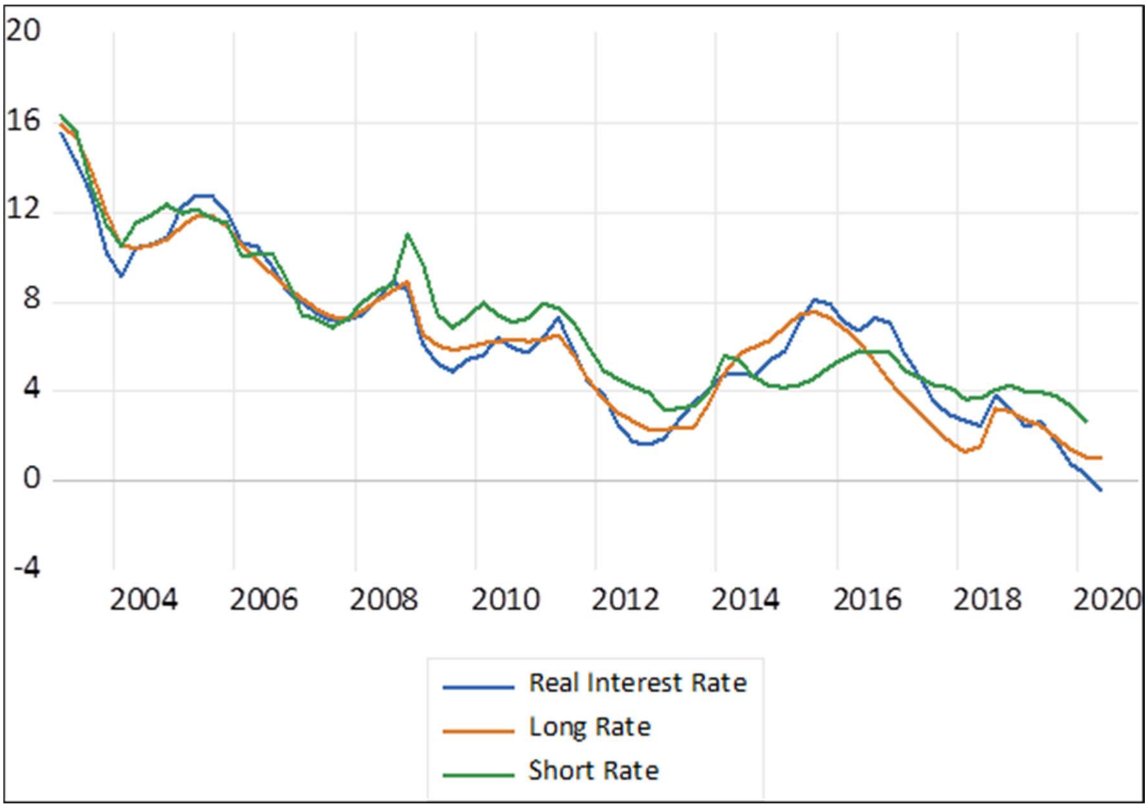
Variables	Coefficients
C	0,0007
	0.0005
Output Gap 1a dif t-1	0,18
	(0,010)*
US Output Gap t-2	-0,12
	(0,057)**
Interest Rate Gap	0,23
	(0,09)**
Govern. Expend Gap	0,002
	(0.007)
Ear Marked Cred. ap t-1	-0,0024
	(0,0009)**
Hiato Cambio Real	-0,0017
	(0.008)
Dummy 03Q2	-0,014
	(0,004)**
Dummy 08Q4	-0,0213
	(0,004)***
Dummy 09Q1	-0,02
	((0,005)***
R2 - ajusted	0,52
F Statistic	9,07

When we plot the effective real interest rate (blue line) with the short-run (green) and long-run interest rate (orange) as show in Figure X, we can notice:

- in 2012, effective real rate was below the short run by a significant amount.
- in this same period, effective real rate was also below the long run, but for a slightly period and magnitude;
- from 2016 until 2018, effective rate was higher than long run interest rate;
- since 2017, effective rate has been significantly below short run rate;
- since 2019, effective rate has been significantly below long run rate.

Based on the long-run estimation of the interest rate, in the second quarter of 2020, the real rate was 1,1% and the short run up to the first quarter 2,7%.

Figure 4 – The Long Short and the Effective Interest Rates



5 – Robustness exercise

The last procedure is not a new methodology to obtain the neutral rate but to compare previous results with an estimated Taylor rule for Brazil.

Table 6 shows a Taylor rule estimated from 2003Q2 to 2020Q1. The inflation term is inflation expectation minus inflation targeting in quarterly basis, and the output gap term is a weighting average between labor market and industrial capacity utilization as described in section 3 (Hiato0620). Both coefficients show significancy and expected signs. A level dummy is introduced called Dumdilma1 during the period of 2011 to 2013 when interest rate was artificially low. Other dummies are included to correct usual outliers.

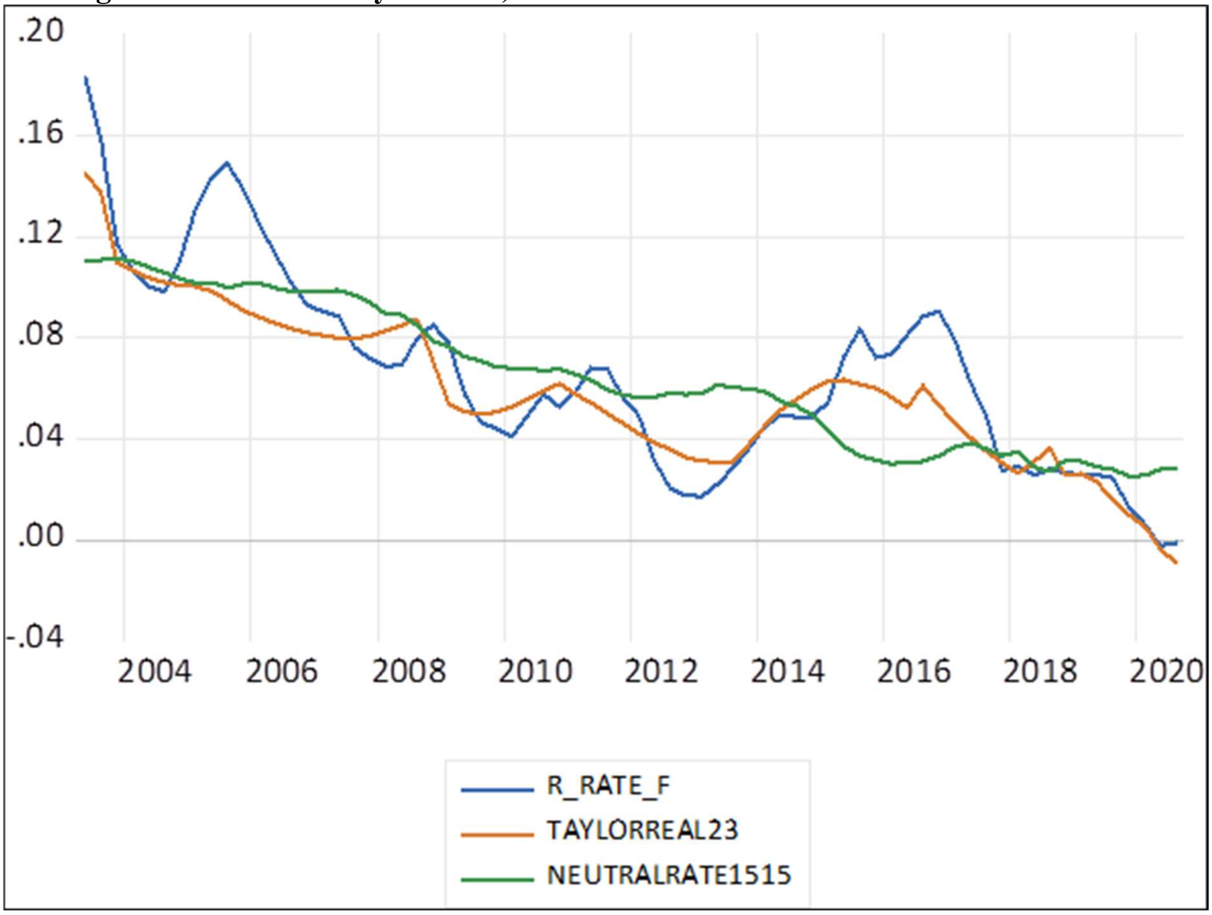
Table 6 Taylor Rule

Variables		Coefficients
C		0,001256
		(0,0004)***
Real Interest Rate t-1		0,914
		(0,023)***
Inf. Exp. -Target		0,23
		(0,076)**
Output		0,033
		(0,01)**
DumDilma1		-0,00154
		(0,0005)***
Dum0304		-0,0052
		(0,001)***
Dum0204		-0,004
		(0,0015)***
Dum0203		-0,0028
		(0,0013)**
Dum0804		-0,0259
		(0,0011)***
Dum1601		0,003
		(0,0014)***
R2 - ajusted		0,98
F Statistic		354,39

When we compared the effective rate with Taylor rule and the average of the Semi-Laubach-Williams rates we can note the following stylized facts as one can see in Figure 5:

- in 2012, effective real rate was below the Taylor rule and from the average rate as well.
- In the period of 2014-2016, effective rate was significantly higher than both rates, which might have aggravated the recession in the period;
- Since 2019, the effective rate is significantly below the average and slightly below the Taylor rate, which means an expansionary monetary policy lately.
- The real interest rate based on the this estimated Taylor rule should be in the 20Q3 at -0.8%.

Figure 5 Estimated Taylor Rule, Neutral and Effective Rate



6- Conclusions

Real interest rates in Brazil are still high in any international comparison, even considered that they have declined significantly in the last few years. The objective of this paper is to measure the equilibrium interest rate in Brazil using different methodologies, including an extension of the Laubach and Williams (2003) using fiscal and credit variables.

The first approach shows us that there is a clear downward trend in interest rate since 1999 based on four technical procedures for Brazil. However, in all of them, in the last period 2015-2019, interest rate in Brazil is still greater than the emerging countries average, but the second Taylor rule (TY2). The latest interest rate estimations based on Taylor rules are negative.

Based on the second approach, the semi-Laubach-Williams, the long run equilibrium rate is in the range of 2-3% depending on the output gap and risk scenario. Our sensitivity analysis has shown that our results changed slightly for different scenarios for Brazil risk premium but deeply regarding potential GDP growth. Considering the alternative scenario for output gap, real rate values are much lower, because in this case output gap is much wider.

Regarding the third approach, on the long-run estimation of the interest rate in the second quarter of 2020, the real rate was 1.1% and the short run up to the first quarter is 2.7%, values very similar to ones found in the second approach.

Using the estimated Taylor rule, we can notice that since 2019 the effective rate is significantly below the neutral rate and slightly below the Taylor rate, which means an expansionary monetary policy lately. The real interest rate based on the this estimated Taylor rule should be at -0.8% in the 20Q3

Two possible extensions of this paper are: (i) to understand the impact of pandemic health crises in the interest rate through the worsening of fiscal conditions. (ii) A DSGE model to estimate the equilibrium rate.

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