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BRAZILIAN DOMESTIC FIXED INCOME MARKET: DETAILS MATTER

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Brazilian Domestic Fixed Income Market: details matter

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ABSTRACT

This text examines how Brazil's domestic fixed-income market prices pre-fixed versus inflation-indexed zero-coupon instruments. It begins by outlining the non-arbitrage principle under which market participants form expectations of real interest rates and implied inflation, showing why fixed-rate (nominal) and IPCA-linked bonds largely track each other's yields yet diverge above certain thresholds. Historical yield data reveal changing slopes in real versus nominal rates and in implied inflation versus nominal rates, suggesting a nonlinear relationship. The author proposes viewing part of the inflation-indexed bond's premium as a "virtual put," which helps explain asymmetric behavior when nominal rates rise or fall. In doing so, the analysis clarifies why these instruments offer partial protection in stressed scenarios—yet also lag in more favorable periods—and highlights that implied inflation often exceeds standard inflation forecasts. The discussion underscores the importance of recognizing such embedded nonlinearities for investors, advisors, and risk managers in the fixed-income arena.

Key-words: Interest rates, Real interest rates, breakeven inflation.

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1. INTRODUCTION

The domestic Fixed Income (FI) market offers various instruments: Government Bonds, Bank Certificates of Deposit (CDBs), Debentures, Real Estate Credit Bills (LCIs), Agribusiness Credit Bills (LCAs), Real Estate Receivables Certificates (CRIs), Agribusiness Receivables Certificates (CRAs), etc.

A fixed-income asset (FI) has the following main parameters: credit risk, repayment flow, issuance rate, and index. Of course, there can be many combinations and increased pricing complexity as more features (such as guarantees) come into play.

Hence, to determine the price of an asset in this class, several parameters need to be provided.

My focus at this time is just comparing a zero-coupon fixed-rate instrument with a zero-coupon instrument indexed to the IPCA (Brazil's consumer price index).

2. NON-ARBITRAGE PRINCIPLE IN THE USUAL APPROACH

Consider all publicly available information regarding inflation expectations, real interest rate expectations, Selic rate expectations, growth expectations, and future exchange rate expectations. In principle, the entire history of these expectations — as well as the variables themselves — is reflected in the market prices freely negotiated out there.

Specifically, assume there is a fixed-rate bond with a 2-year maturity that pays its entire amount at maturity (for instance, an LTN—National Treasury Bill), and another bond indexed to the IPCA, with all identical features except the index. For the two bonds to be equivalent at maturity, let's assume:

$$E_t^{informação}[(1+j)^T] = E_t^{informação}\left[\prod_{i=t_1}^{t_n}(1+\pi_t)^i\,(1+y_t)^i\right];\ \sum t_j\ = T \qquad (1)$$

The above only establishes that the expected total return on the fixed-rate asset (the left-hand side) equals the expected accumulated inflation plus real interest over the same period. In other words, the conditional expected values, given the available information, are equal. This is one possible model, and as we will see later, there is an alternative way to approach it.

This expectation seems logical because, if there were a discrepancy, it would mean that one instrument is cheaper than the other. In that case, market participants would buy the cheaper asset and sell the more expensive one until the equilibrium is restored.

Notice that the indexation on the right side of (1) happens because inflation varies each period, as does the real interest rate for that period, and summing up all sub-intervals must yield the total period.

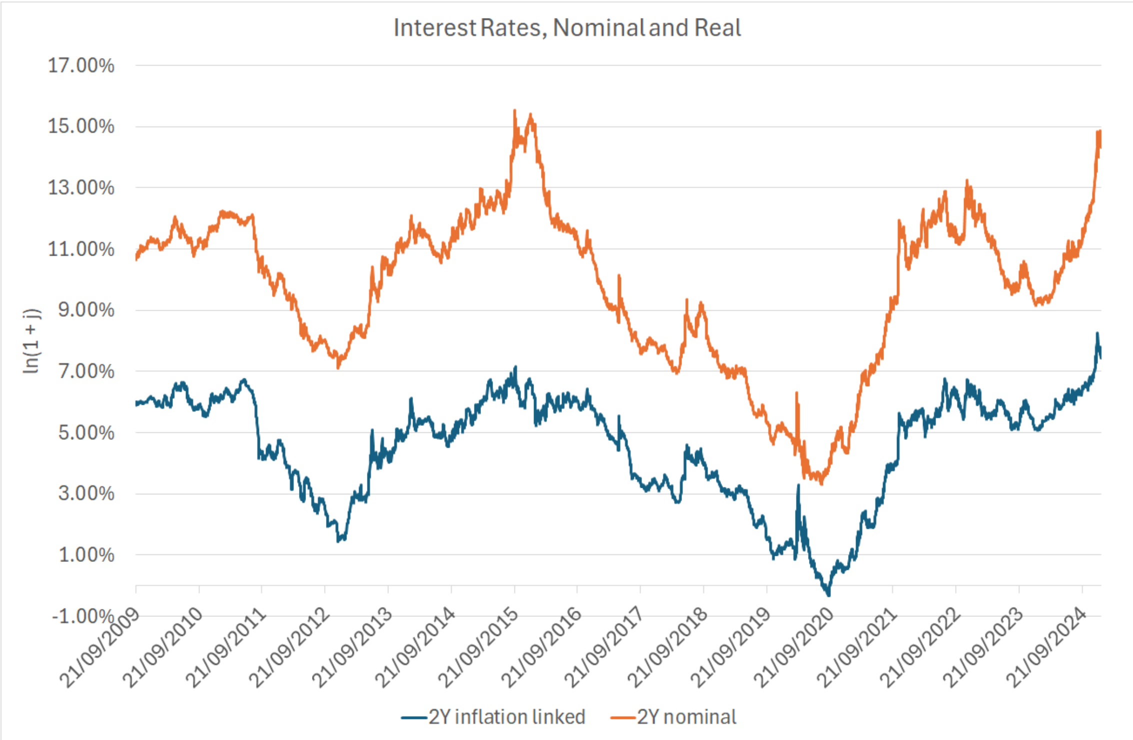
A big simplification is to assume, in the previous equation, only two variables in the second term and replace the expectation equations with deterministic equations, meaning that at any point in time, there is no arbitrage:

$$[(1+j)^T] = [(1+y)^T(1+\pi)^T] \therefore \ln(1+j) = \ln(1+y) + \ln\,(1+\pi) \qquad (2)$$

In particular, the market negotiates the values of "j" and "y," making it possible to estimate the remaining term, which is called "breakeven inflation" or "implied inflation." For example, daily, Anbima publishes these values on its website in its closing yield curves at the link: Curvas de Juros - Fechamento – ANBIMA.

3. THE SOLUTION

Let's consider the historical series of fixed-rate and IPCA-indexed interest rates:

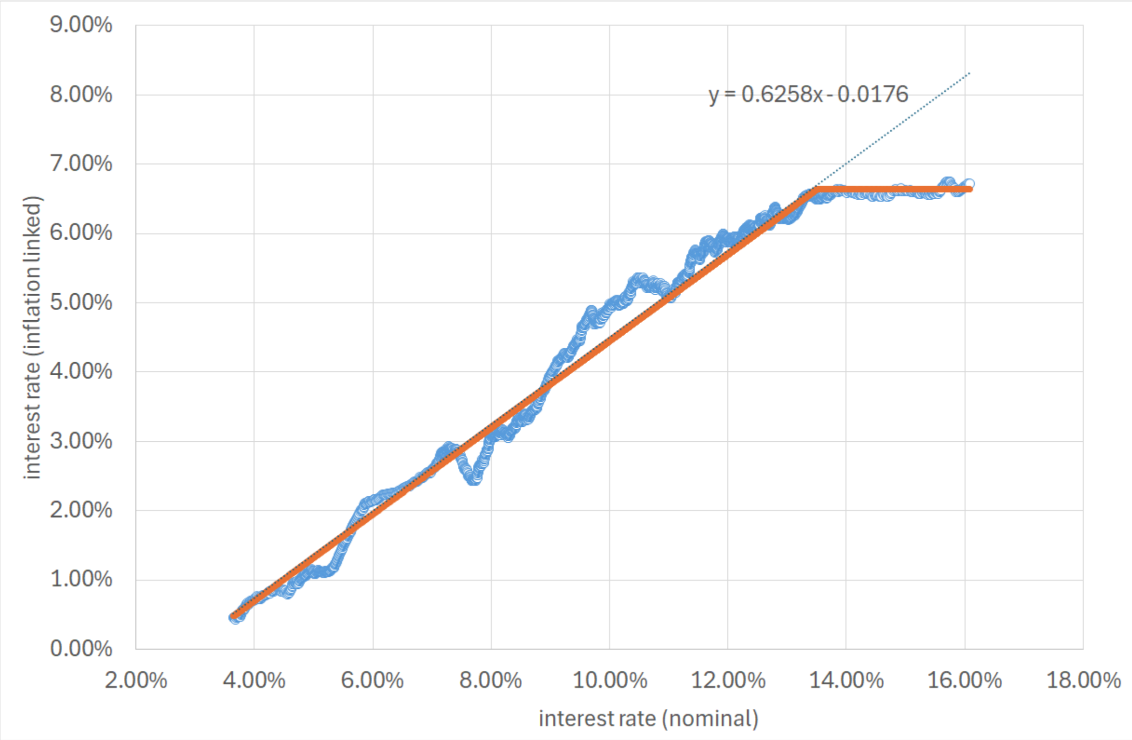


Looking at the figure above, in general, the correlation between both interest rates is high. However, one notices that the gap between these rates changes over time: sometimes they come closer, other times they drift apart. Working with the idea from the previous equation, one obtains a very useful approximate relationship between the slopes of the real rate and of implied inflation, relative to the nominal rate:

$$\Delta[ln(1+j)] = \Delta[ln(1+\pi)] + \Delta[ln(1+y)] \therefore 1 = \frac{\Delta[ln(1+\pi)]}{\Delta[ln(1+j)]} + \frac{\Delta[ln(1+y)]}{\Delta[ln(1+j)]} \quad (3)$$

4. THE RELATIONSHIP IS NOT LINEAR

Now we need to construct the curves. Ordering the data by the level of the fixed rate and smoothing them by a moving average yields the following figures, which respectively show the real interest rate as a function of the nominal rate, and the implied inflation as a function of the nominal rate:

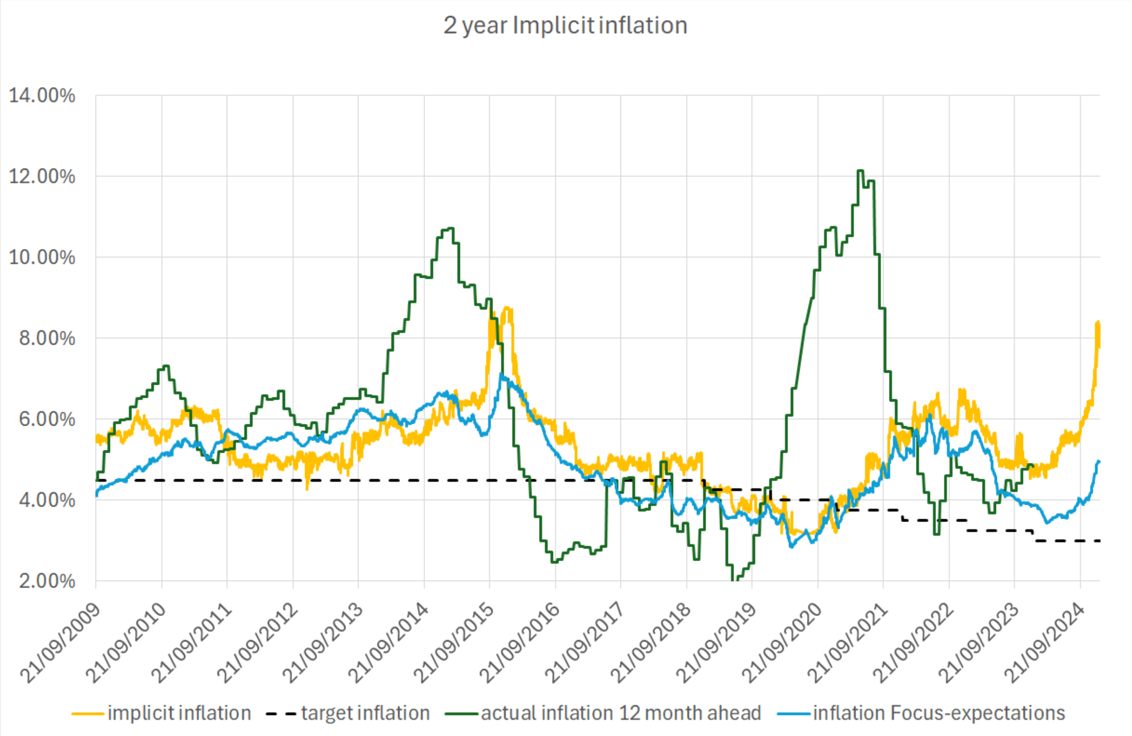


Even though, for much of the observed range, the real/nominal slope is around 62% (with some deviation), starting from the neighborhood of 12% or 13%, there is a significant reduction in slope, which drops to around 15%. Meanwhile, in the case of implied inflation, as expected, for most of the observed range of fixed rates, its slope is about 33%, but from nominal rates around 13%, it speeds up to roughly 82%!

$$1 = \beta_\pi(j) + \beta_y(j) + \varepsilon \qquad (4)$$

5. A LIGHTNING BOLT IN A BLUE SKY

Looking at the traded implied inflation values from the same dataset, we get:



From the figure above, implied inflation is always above the 12-month-ahead target and more often above the Focus survey's 12-month-ahead expectations. Is that a bias? Are we missing a term in the equilibrium equation?

My conjecture is that you can reframe the equilibrium equation by introducing an additional term whose payoff is akin to an out-of-the-money put on the fixed-rate asset. In other words, when one buys an indexed asset, part of the value assigned to what is called "implied inflation" is the premium of a (virtual) put option on the fixed-rate bond. The strike varies over time, and it seems to be related to the real rate level considered "bearable".

The existence of this virtual put explains why fixed-rate and indexed bonds track each other up to a certain point and then diverge (as seen in the previous section chart). It also explains why fixed-rate bonds tend to do better in calmer times (because the cost of that put is embedded in the indexed assets).

6. CONCLUSION

Note that I am not recommending any specific index in the Fixed Income segment, whether CDI, fixed rate, or inflation linked. I merely presented an analysis of a topic that, in the fascinating world of Fixed Income, is often overlooked or unknown.

I believe that a significant portion of economic agents operating in this segment—be they investors, advisors, traders, portfolio managers, risk officers—might not consider these nonlinearities and thus implicitly assume the relationship is that given by a linear regression.

To illustrate the importance of this subject, just look at the public debt, specifically federal government bonds. The sum of these securities amounts to something about 55% of the total volume in the public hands.

From the investor's perspective, the decision happens in two steps:

1. **Do I want interest-rate risk or not**? (i.e., do I choose CDI-linked or not?)
2. **If not CDI**, do I prefer a fixed-rate security or an inflation-linked one?

From a risk standpoint, the fact that implied inflation's slope jumps brutally once nominal rates reach a certain threshold suggests that these indexed securities contain a "protection" during nominal-rate stress scenarios. However, this protection is only more effective above a certain level of the fixed nominal rates. Concretely, when nominal rates go up (say from 14% to 15%), real rates only rise about 0.15%, not 1%. Thus, losses on the indexed securities are on the order of 15% of the losses on fixed-rate assets.

From a potential return standpoint, in symmetrical fashion, these same securities that can protect against the upward stress of nominal rates (as observed in recent months) will be slower to respond to any drop in those nominal fixed rates. This happens because, at that high nominal-rate level, much of the nominal rate's decline is reflected in a decline in implied inflation, rather than a decline in real rates. Concretely, if stressed nominal rates fall (say from 15% to 14%), real rates drop by only about 0.15%, not 1%, so gains for the indexed bonds amount to about 15% of the gains on fixed-rate bonds.

Observe that the presence of the virtual put I suggested is consistent with the stylized behavior in those two paragraphs above.

Finally, the frequent spread between implied inflation and inflation expectations can be interpreted as the price of that virtual put.