Betting Against Beta in Brazil

Unlike assumed by the CAPM, leverage and margin constraints prevent agents from simply leveraging or de-leveraging their optimal allocations to reach their risk appetite. As a consequence, agents directly buy riskier securities to match their desired risk levels, creating an asymmetry in the risk-adjusted returns required for lower-beta assets versus high-beta ones. This study aims at analyzing the dynamics of the betting against beta factor, a market-neutral, self-financed portfolio built to arbitrage the beta anomaly in the Brazilian equity market.

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Vinicius Esposito[i]

The better part of modern portfolio theory operates on the capital asset pricing model (CAPM) universe. This so-called CAPM world has many underlying assumptions, notably that: (i) investors agree on the probability distributions of assets' end-of-period values, (ii) that these common distributions are at least joint stable with a single characteristic component (or joint normal), (iii) agents maximize their expected utility, whose function is increasing on wealth with diminishing marginal returns, (iv) investors may take positions of any size in any asset and they may borrow or lend any sum at the risk-free rate of interest.

Arguably, two of these assumption have suffered more criticism than the others. Firstly, assumption (ii) has been called into question due to the now reasonably established fact that asset returns, especially those of stocks, have fat-tailed distributions. Consequently, rare events are heavily underestimated by the employment of a distribution such as the Gaussian. However, the aim of this study is to investigate the restrictiveness of assumption (iv), building on the insights of Black (1972) and, more recently, Frazzini and Pedersen (2014).

Investors, as opposed to what is posited by the CAPM universe under assumption (iv), are constrained in the leverage that they can take. This means that, instead of investing in the portfolio with the best reward for risk (i.e. highest Sharpe ratio) and then leveraging or de-leveraging to suit their risk appetite, investors instead directly buy risky securities to match their risk preferences. This is very often the case of pension and mutual funds which often have leverage ceilings, or the case of individuals who simply lack access to funding that enables them to scale their positions as they wish. As Frazzini and Pedersen (2014) note, the recent demand for exchange-traded funds (ETFs) with embedded leverage is a symptom of such phenomenon.

Therefore, this preference towards high-risk assets implies that they require lower risk-adjusted returns than low-risk assets. In the CAPM geography, this suggests a flatter than expected security market line (Black, Jensen, and Scholes, 1972) and that a restricted borrowing assumption instead of assumption (iv) should prove a better fit for empirical data (Black, 1972, 1993).

One way to study the asset pricing impact of this beta anomaly is to analyze the betting against beta (BAB) factor, which is a portfolio that is long low-beta assets and short high-beta ones. This factor achieves market-neutrality by leveraging the first leg of the portfolio to a beta of 1 and de-leveraging the second leg to a beta of 1. Frazzini and Pedersen (2014) examine the behavior of the BAB factor using a model with agents of different leverage constraints. Analyzing data from 20 international stock markets, Treasury bond markets, credit markets, and future markets, they not only find evidence of consistent BAB returns, but they encounter a much flatter security market line than is predicted by the CAPM.

This study examines the return dynamics of the BAB factor in the Brazilian stock market, which could provide interesting insights considering that developing markets are expected to face harsher borrowing constraints than the markets examined by Frazzini and Pedersen (2014).

Data

The data in this study are collected from two main sources. The sample of Brazilian stock prices is retrieved from the Economática database, a widely used platform for Latin American financial data. The Brazilian equity data used includes all stocks traded on the Bovespa exchange from January 2000 to April 2017. On the other hand, the riskless rate of return, the market risk premium and other factors are supplied by the Brazilian Center for Research in Financial Economics (NEFIN)[1] belonging to the University of São Paulo. NEFIN provides daily returns to the risk-free rate, the market risk premium, as well as the Fama-French (1992, 1993) value and size portfolios, the Carhart (1997) momentum factor, and the Pastor and Stambaugh (2003) liquidity factor. Excess returns are computed in excess of the 30-day DI swap rate retrieved from NEFIN and all returns are in local currency.

[1] See www.nefin.com.br.

Creating the Betting Against Beta Factor

Following Frazzini and Pedersen (2014), the first step is estimating ex-ante betas using rolling regressions. Daily data is used since a larger sample frequency improves the covariance estimation (Merton, 1980). The beta, as usual, is the coefficient from a regression of a stock's excess returns on the market risk premium, and given by:

$$\hat{\beta}_i^{TS} = {\hat{\sigma}_{i,m}}/{\hat{\sigma}_m^2},$$

where $\hat{\sigma}_{i,m}$ is the estimated covariance between the stock and the market, $\hat{\sigma}_m^2$ is the estimated variance of the market return, both calculated using one-year rolling windows (which results in the loss of the first year of the sample). Betas are estimated only if there are at least 120 days of non-missing data in this yearly window. It is worth noting that it is standard procedure in the literature to compute betas using correlation and volatilities, accounting for the fact that correlations seem to move more slowly than volatilities. This requires using longer time horizons to estimate the correlations, which would, in turn, reduce even more the sample. In order to preserve the sample size and for simplicity, the beta components in this study are all computed using yearly windows.

Furthermore, the approach of Vasicek (1973) is used to mitigate the influence of outliers and shrink the time series estimates of betas toward their cross sectional mean. That is,

$$\hat{\beta}_i \;= w_i\hat{\beta}_i^{TS} + (1 - w_i)\hat{\beta}_i^{XS}.$$

Vasicek (1973) uses a Bayesian shrinkage factor that emphasizes the time series estimate when the estimates have lower variance or when the cross-sectional variance is greater. In this study, a constant shrinkage factor is used and set to $w_i = 0.6$, which shouldn't change results in a significant way and is common practice in the literature. The cross-sectional mean beta is also taken as constant and set to $\hat{\beta}_i^{XS} = 1$, given that there are inherent measurement errors in the regressions as the real market portfolio is not observable. It is important to note that this choice of shrinkage factor will not influence the sorting of securities into beta portfolios, but will, however, affect the construction of the BAB portfolios, since the betas are used to scale the long and short legs of the BAB factor portfolio.

The first step to build the betting against beta factor is ranking all assets according to their estimated betas every time period, and then assigning securities to two portfolios, the high- and low-beta. The high-beta portfolio is the one with all stocks with beta larger than the median at a given time, while the low-beta holds all stocks with betas below the median. These rankings are recalculated at the beginning of each calendar month using the data from the immediately prior month (i.e. monthly rebalancing).

Again, following Frazzini and Pedersen (2014), portfolio weights should tilt the high-beta (low-beta) portfolio towards stocks with the highest (lowest) betas. Therefore, the weight of a stock i in period t is given by:

$$W_i = 2 \times \frac{\left|rank_i - \overline{rank}\right|}{\sum_{i=1}^{n}\left|rank_i - \overline{rank}\right|},$$

where $rank_i$ is the beta-rank of stock i among all n assets traded at time period t-1. $\overline{rank}$ is the mean rank. By construction, weights sum to one at the formation time ($\mathbf{1}_n' W_H = 1$ and $\mathbf{1}_n' W_L = 1$).

In order to build the BAB factor, both portfolios (high- and low-beta) are rescaled to have a beta of one at portfolio. Thus, the BAB is a self-financing (as it shorts the risk-free rate) zero-beta portfolio, defined by:

$$r_{t+1}^{BAB} = \frac{1}{\beta_t^L}\left(r_{t+1}^L - r_f\right) - \frac{1}{\beta_t^H}\left(r_{t+1}^H - r_f\right),$$

where β_t^L (β_t^H) is the weighted average beta of the low-beta (high-beta) portfolio, r_{t+1}^L (r_{t+1}^H) is the weighted return on the low-beta (high-beta) portfolio. r_f is the riskless rate of return.

The performance of the BAB factor and beta-sorted portfolios are examined through calendar-time regression of the portfolios excess returns using the CAPM, 3-factor, 4-factor and 5-factor models as follows:

$$R_{it}^e = \alpha_i + \sum_{k=1}^{m} \beta_{ik} R_{kt} + u_{it},$$

where R_{it}^e is the excess-return of portfolio i on period t, R_{mt} is the factor excess return and m is the number of factors. The independent variables of these regression are defined according to each model:

a) CAPM: market risk premium (MKT – Rf).
b) Three-factor: CAPM expanded with Fama and French's (1992, 1993) value (HML) and size (SMB) portfolios.
c) Four-factor: three-factor model expanded with Carhart's (1997) momentum factor (WML).
d) Five-factor: four-factor model expanded with Pastor and Stambaugh's (2003) liquidity factor.

Before delving into the results of the aforementioned regressions, one can take a simpler approach to estimate the dynamics of a BAB strategy. Assume it is possible (and plausible) to relate Sharpe ratio to beta using the relation implied by Table 2 ($sharpe_i = \mu_i/\sigma_i = -0.6\beta_i + 0.3$)[2] and choose any two portfolios such that $\beta_1 < \beta_2$. In order for both have the same risk: $x_1\sigma_1 = x_2\sigma_2$, where x_i is the sum invested in asset i and $x_1 + x_2 = 1$. The return of a portfolio that

[2] A more general hypothesis would be to assume the Sharpe ratio is a linear function of beta, $SR_i = f(\beta_i)$. The return of the long-short portfolio is then:

$$E(r_t^{LS}) = \frac{df}{d\beta} x_1\sigma_1(\beta_1 - \beta_2) - (2x_1 - 1)r_f.$$

Yet, the analysis remains valid as long as $f'(\beta_i) < 0$, that is, high-beta assets or portfolios have lower risk-adjusted returns than low-beta assets, which is true in the Frazzini and Pedersen's (2014) framework and is assumed to hold in this study.

is long the low-risk portfolio and short the high-risk one is thus given by:

$$E(r_t^{LS}) = 0.6x_1\sigma_1(\beta_2 - \beta_1) - (2x_1 - 1)r_f$$

Setting these portfolios equal to the low- and high-beta portfolios created using the methodology previously described and using time series averages, the return of the BAB strategy sits around 2% a year. Furthermore, for every R$1.00 long the low-beta portfolio, R$0.60 is short the high-beta portfolio and is R$0.40 borrowed at the risk-free rate.

Findings and Discussion

The distribution of shrunk betas over time is shown in Figure 1, where stocks are, each month, sorted into deciles according to their betas. Decile portfolios are equally weighted. There is a substantial cross-sectional variation among betas in the Brazilian stock market, as well as a lot of time variation of these coefficients. As expected, the betas suffer a compression around the year 2008 as a result of the global financial constraint, however, this compression seems to be relatively mild and quickly dissolves. Notably, lower percentiles exhibit lower variation over time, a behavior also documented by much of the preceding literature.

Table 2 reports returns and other aspects of the portfolios formed on their *ex-ante* betas. The same ten beta-sorted portfolios and the BAB factor are considered. As proposed, the BAB portfolio carries positive returns, risk-adjusted or not, with virtually no market exposure. In accordance to Black (1972) and Frazzini and Pedersen (2014), the average returns of the beta portfolios are close to each other, which means the Brazilian security market line is also relatively flat. One possible reason for that is the existence of short-sale friction as proposed by Brent et al. (1990), according to whom high-beta assets are more likely to be expensive to sell short. These frictions may be even stronger in the Brazilian market where short-selling is more restricted than it is in developed markets.

The alphas decline almost monotonically from low-beta to high-beta portfolios, a phenomenon which is not only limited to the CAPM but seems to be robust to all factor models estimated. Interestingly, high-beta portfolios have significant negative alphas, with the highest decile portfolio having more than 1% of alpha cost regardless of the model estimated. Analysis of Sharpe ratio for the Brazilian market is limited due to them being negative across nearly all beta portfolios. Yet, a downward trend from the low- to the high-beta clusters is evident.

The rightmost column of Table 2 shows returns of the betting against beta factor. The BAB factor portfolio has considerable alpha across all models. However, it is only statistically significant at 5% when estimated in a the three-factor model. The difficulty in rejecting the null hypothesis of zero return likely lies in the small number of stocks in the sample, as well as the short time horizon available for analysis, a problem recurrent in the Brazilian financial literature.

Overall, these results point to the fact that the security market remains too flat in the Brazilian equity universe, as it seems to do in most other markets. Additionally, in a country of negative the market-neutral BAB factor portfolio delivers a relatively high reward for risk.

Conclusion

The betting against beta factor differs from any of its competing risk factors in that it arises from Fischer Black's theoretical exercise of questioning one the CAPM's basic assumptions. Hence, not only does it have a compelling economic narrative behind it, but it also seems to have the data to back its case. This short study aimed at providing a simple analysis of the application and dynamics of such a strategy in the Brazilian equity market.

Consistent with the literature, it is found that portfolios of higher-beta assets deliver lower alphas and Sharpe ratios than those of low-beta assets. Also, the Brazilian security market line fails to follow the behavior predicted by the CAPM and instead adopts its international trend of being too flat.

Thus, the application of beta factors such as Frazzini and Pedersen's (2014) in Brazil appears to be a significant way to explore this anomaly of the CAPM. On one hand, the sample of stocks in Brazil is very restricted when compared to that of developed markets, on both the cross-section and the time series dimensions. This significantly affects any model estimation and may be one of the reasons the BAB factor's alpha is hardly significant across nearly all of the models estimated in this short article. However, even though it is not attempted to clarify whether beta-arbitrage is true alpha or a mere exposition to unknown sources of risk, the reward for risk perspective of betting against beta in Brazil stands promising.

[i] **Vinicius Esposito; GV Invest, São Paulo School of Economics (Fundação Getulio Vargas).**

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Table 1: Summary of equity data

Year	Average number of firms	Average monthly market return (%)	Average monthly risk-free rate (%)
2001	235	-1.24	1.39
2002	231	0.33	1.48
2003	238	4.57	1.71
2004	253	2.00	1.24
2005	264	2.39	1.44
2006	266	2.32	1.14
2007	309	2.95	0.91
2008	334	-4.11	0.96
2009	328	4.63	0.76
2010	330	0.46	0.77
2011	333	-0.62	0.91
2012	320	1.02	0.65
2013	312	-0.23	0.64
2014	305	-0.28	0.84
2015	294	-1.06	1.03
2016	282	2.55	1.08
2017	282	2.52	0.91

Table 2: Brazilian equities, 2001-2017

Alpha is the intercept in a regression of monthly excess returns from January 2001 to April 2017. Returns and alphas are in monthly percent, *t*-statistics are shown below coefficient estimates, and 5% statistical significance is indicated in bold. Volatility is annualized, Sharpe ratio is not due to the issues resulting from annualizing it (Lo, 2002).

	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	BAB
	(low beta)									(high beta)	
Excess return	-0.21	0.05	0.06	-0.08	-0.20	-0.13	-0.81	-0.59	-0.57	**-1.45**	0.68
	(-0.51)	(0.12)	(0.15)	(-0.17)	(-0.42)	(-0.25)	(-1.58)	(-1.04)	(-0.96)	(-1.96)	(1.7)
CAPM alpha	-0.27	-0.03	-0.02	-0.17	-0.31	-0.24	**-0.92**	**-0.72**	**-0.72**	**-1.62**	0.68
	(-0.81)	(-0.11)	(-0.08)	(-0.55)	(-0.98)	(-0.79)	(-3.32)	(-2.6)	(-2.4)	(-4.02)	(1.69)
3-factor alpha	-0.18	0.07	0.07	-0.02	-0.19	-0.17	**-0.83**	**-0.67**	-0.69	**-1.63**	**0.81**
	(-0.59)	(0.25)	(0.27)	(-0.08)	(-0.78)	(-0.64)	(-3.62)	(-2.77)	(-2.75)	(-4.96)	(2.07)
4-factor alpha	-0.28	0.00	0.02	-0.07	-0.13	-0.19	**-0.85**	**-0.60**	**-0.42**	**-1.19**	0.48
	(-0.87)	(-0.01)	(0.07)	(-0.27)	(-0.51)	(-0.7)	(-3.64)	(-2.46)	(-1.78)	(-4.03)	(1.28)
5-factor alpha	-0.28	0.01	0.01	-0.06	-0.11	-0.19	**-0.85**	**-0.60**	**-0.45**	**-1.25**	0.52
	(-0.89)	(0.03)	(0.06)	(-0.21)	(-0.43)	(-0.71)	(-3.63)	(-2.43)	(-1.94)	(-4.39)	(1.37)
Beta (ex-ante)	0.46	0.58	0.65	0.70	0.76	0.82	0.90	0.98	1.08	1.28	0.00
Beta (realized)	0.45	0.59	0.61	0.65	0.74	0.83	0.86	1.01	1.05	1.24	-0.01
Volatility	18.85%	19.46%	20.04%	21.02%	23.05%	23.86%	24.00%	26.53%	28.02%	34.69%	18.85%
Sharpe ratio	-0.13	0.03	0.04	-0.04	-0.11	-0.06	-0.40	-0.27	-0.25	-0.50	0.43

Figure 1: Beta sorted portfolios across time

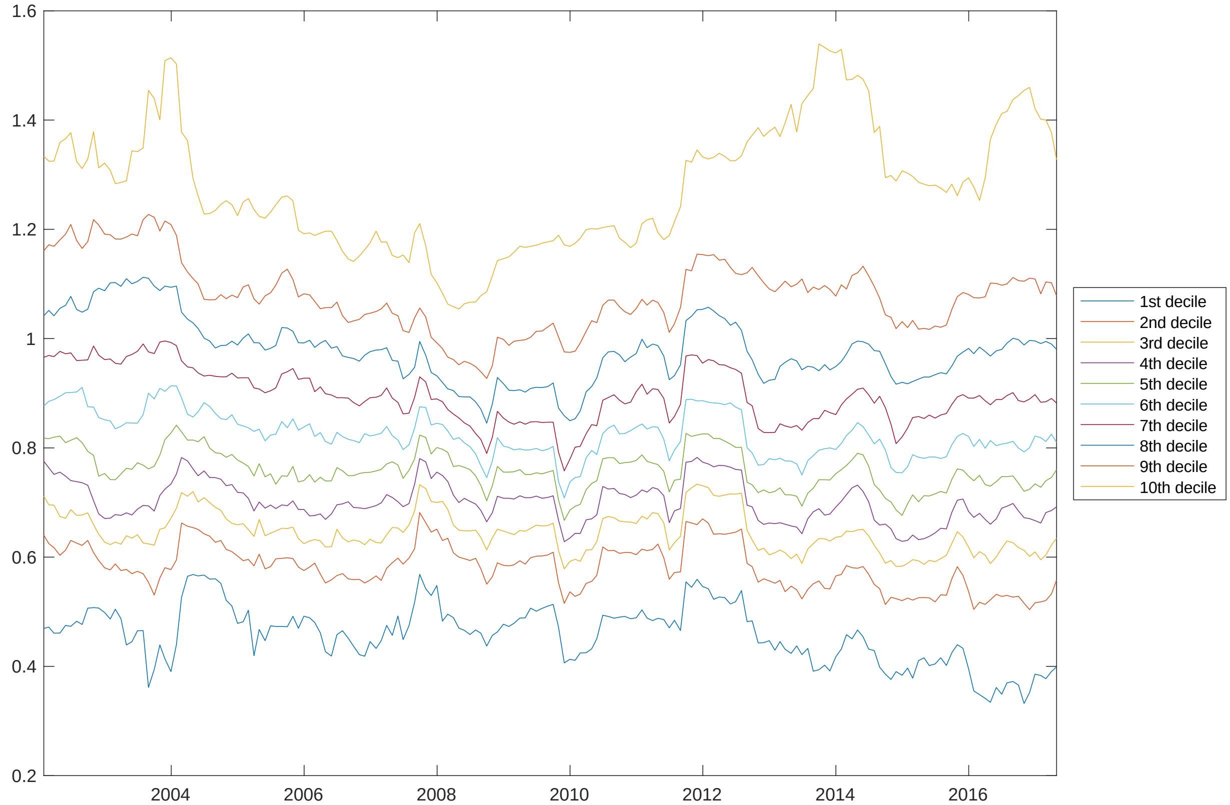

Figure 2: Cumulative log factor returns