In the last decade, the field of monetary economics has undergone a kind of forced revolution. The financial crisis has induced the central banks of almost all the major developed countries to reduce interest rates to a level near or even below zero. Unconventional policies, such as quantitative easing, were introduced and the interest rates were kept low, almost fixed, for a long time.

This scenario led to new questions regarding the means and ends of monetary policy. Eventually, such questions generated important research on the field. This short study aims to clarify an important puzzle that emerged throughout the past years and that was emphasized by Cochrane (2016): the stability of inflation even when interest rates seem to be unresponsive.

The Empirical Puzzle

Interest rates have been at the zero lower bound in the United States and Europe for years after the Great Recession. Additionally, in Japan, interest rates have been stuck at low values since the 1990s. Meanwhile, inflation has been relatively stable in the three monetary areas. There were some swings in the inflation rate such as the worldwide disinflation post-2008, but inflation/deflation has not spiralled out of control: it has come back to the same level of before the recession.

Past inflation stability may be attributed to the central banks unconventional policies. However, Cochrane (2016) points that conventional Keynesian theories cannot fully explain such stability when interest rates were unresponsive.

The crucial stability and determinacy condition of inflation in conventional Keynesian models is the Taylor principle: central banks should answer shocks in inflation with greater swings in the nominal interest rate. The stubbornness of near-zero interest rates in the recent past shows that the
Taylor principle was not applied. Nevertheless, inflation has remained stable.

This study will try to explain concisely the Cochrane’s approach of showing this empirical puzzle. The first part will explain the simple model used to replicate the Keynesian framework. The second part will focus on the results when adaptive expectations are present. The third part will focus on the more common and sophisticated results when rational expectations are introduced to the model. Finally, the fourth part will summarize the analysis.

**Benchmark model**

Most of the Keynesian models have their inflation and output equilibria and dynamics defined by the following equations:

\[ i_t = r_t + \pi_t^e \]  
\[ \pi_t = \pi_t^e + \kappa x_t \]  
\[ x_t = -\sigma(r_t - v_t^i) \]  
\[ i_t = \phi \pi_t + v_t^i \]

Where \( t \) is the time subscript; \( i_t \) is the nominal interest rate; \( r_t \) is the real interest rate; \( \pi_t \) is the inflation rate; \( \pi_t^e \) is expected inflation; \( x_t \) is the output gap; \( v_t^i \) is the real interest rate shock; \( v_t^i \) is the monetary policy shock; \( \phi \) represents the responsiveness of monetary policy to inflation; and \( \sigma, \kappa \) are positive parameters.\(^1\)

These equations are usually derived from agents’ behaviour, price and/or wage stickiness and no-arbitrage conditions. Different studies tend to derive them from different conditions. However, both the qualitative results and the dynamics of the system are not so distinct when comparing mainstream models. Cochrane (2016) abstains from using such derivation to increase the generality of his critique, which seems to favour his argument.

Such system can be solved to determine inflation in relation to the shocks and the expected inflation. The result, below, can be further used to analyse the dynamics and equilibrium of inflation when expected inflation is also determined.

\[ \pi_t = \left( \frac{1 + \sigma \kappa}{1 + \phi \sigma \kappa} \right) \pi_{t-1}^e - \left( \frac{\sigma \kappa}{1 + \phi \sigma \kappa} \right) (v_t^i - v_t^i) \]

**Adaptive expectations**

First and foremost, let us analyse the implications of this system when adaptive expectations are assumed. (\( \pi_t^e = \pi_{t-1} \)) In other words, the implications of different values of \( \phi \) for the determinacy and stability of inflation in the old-Keynesian framework. Clearly:

\[ \pi_t = \left( \frac{1 + \sigma \kappa}{1 + \phi \sigma \kappa} \right) \pi_{t-1} - \left( \frac{\sigma \kappa}{1 + \phi \sigma \kappa} \right) (v_t^i - v_t^i) \]

When monetary policy adheres to the Taylor principle, \( \phi > 1 \) and \( \left( \frac{1 + \sigma \kappa}{1 + \phi \sigma \kappa} \right) < 1 \). This causes inflation to slowly return to its steady-state equilibrium whenever there is a temporary shock.

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\(^1\) For simplicity, we will keep Cochrane’s notation to help the reader that wants to dive deeper into the research.

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When $\phi < 1$, however, $\left(\frac{1+\sigma K}{1+\phi \sigma K}\right) > 1$. So, even when inflation starts at the steady-state value, every shock will lead to inflation spiralling to infinite inflation or deflation. Thus, the model is unstable.

If someone had this model in mind when interest rates were approaching the zero-lower bound, the only possible prediction he/she could make would be a spiralling deflation if the zero-lower bound became binding. The straightforward reason is that at the zero-lower bound, interest rates could not be reduced more than inflation: $\phi < 1$. Therefore, considering the recent past, the stable inflation is clearly a puzzle when the old-Keynesian framework is used.

### Rational expectations

Using adaptive expectations is outdated and does not reflect the recent advances in economics. A more conventional and robust approach would be to introduce rational expectations. Equivalently, agents rationalize their expectations in accordance with the behaviour of the economy ($\pi_t = E_t \pi_{t+1}$): 

$$
\pi_t = \left(\frac{1+\sigma K}{1+\phi \sigma K}\right) E_t \pi_{t+1} - \left(\frac{\sigma K}{1+\phi \sigma K}\right) (v_t^i - v_t^r)
$$

Using the same result to substitute out $\pi_{t+1}$, we get:

$$
\pi_t = -\left(\frac{\sigma K}{1+\phi \sigma K}\right) (v_t^i - v_t^r) - \left(\frac{\sigma K}{1+\phi \sigma K}\right) \left(\frac{1+\sigma K}{1+\phi \sigma K}\right) (v_{t+1}^i - v_{t+1}^r) + \left(\frac{1+\sigma K}{1+\phi \sigma K}\right)^2 E_t \pi_{t+2}
$$

Then, we can reproduce this process indefinitely, finding a pattern:

$$
\pi_t = -\left(\frac{\sigma K}{1+\phi \sigma K}\right) \sum_{j=0}^{\infty} \left(\frac{1+\sigma K}{1+\phi \sigma K}\right)^j E_t (v_{t+j}^i - v_{t+j}^r) + \lim_{T \to \infty} \left(\frac{1+\sigma K}{1+\phi \sigma K}\right)^T E_t \pi_{t+T}
$$

When $\phi > 1$, $\left(\frac{1+\sigma K}{1+\phi \sigma K}\right) < 1$ and inflation is non-exploding ($\lim_{T \to \infty} \left(\frac{1+\sigma K}{1+\phi \sigma K}\right)^T E_t \pi_{t+T} = 0$) only if:

$$
\pi_t = -\left(\frac{\sigma K}{1+\phi \sigma K}\right) \sum_{j=0}^{\infty} \left(\frac{1+\sigma K}{1+\phi \sigma K}\right)^j E_t (v_{t+j}^i - v_{t+j}^r)
$$

Therefore, the Taylor principle induces stability and determinacy by forcing exploding paths out of the equilibrium. Such method is different from the old-Keynesian understanding of high interest rates reducing aggregate demand and, then, forcing inflation down. Now, the overreaction of the central bank to inflation leaves the economy two options: exploding inflation or stable inflation determined by the equation above. Spiralling inflation is assumed as undesirable and the model becomes determinate and stable.

When $\phi < 1$, that is not the case. Inflation could take any value and still be consistent with the
model. Anyway, the model is only stable and determinate on expectations, as isolating $E_t \pi_{t+1}$ shows:

$$E_t \pi_{t+1} = \left(1 + \phi \sigma \kappa \right) \pi_t - \left(\frac{\sigma \kappa}{1 + \sigma \kappa}\right) (v^i_t - v^r_t)$$

Thus, we cannot determine the inflation rate, just its expectation. The most we can do is to call $\delta_{t+1} = \pi_{t+1} - E_t \pi_{t+1}$ the forecast error and define:

$$\pi_{t+1} = \left(1 + \phi \sigma \kappa \right) \pi_t - \left(\frac{\sigma \kappa}{1 + \sigma \kappa}\right) (v^i_t - v^r_t) + \delta_{t+1}$$

In sum, the new-Keynesian model with rational expectations and $\phi < 1$ is a stable model, but an indeterminate one, which means something different in the economy should also be affecting inflation. There are multiple equilibria in the model.

Cochrane (2016) shows the problem of such indeterminacy by plotting the graph of possible paths for inflation when there is an expected interest rate shock. For simplicity, we reproduce this graph. The solid lines show the behaviour of inflation when there are no forecast errors in two different situations: one with higher price flexibility (less sticky) and one with less. The important part are the dashed lines. They represent all possible paths for inflation when $\phi < 1$, showing the problems of an indeterminate model.

The more observant reader should also have noticed that the short-run relationship between inflation and interest rates is different than the norm in the case where there are no forecast errors. An interest rate rise will lead to higher inflation both in the short-run and the long-run. This is another interesting point made by the paper: when the Taylor principle cannot be applied, short-run comovement between interest rates and inflation is inverted.

Although this extra point is important for the Cochrane’s defence of the fiscal theory of the price level, a treatment of the subject is out of the scope of this short study. The main objective is to expose what we believe to be the most important argument of Cochrane (2016): that conventional monetary theory cannot explain the coexistence of stable inflation and monetary policy that is all but an interest rate peg.

Concluding Remarks

The aftermath of the Great Recession has left us with an almost natural experiment. We could observe inflation stability despite unresponsive interest rates. Cochrane (2016) tries to use this

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experiment to test widely-understood conventional Keynesian theories. The conclusion is that neither the Old-Keynesian nor the new-Keynesian theories can explain this empirical combination of stable inflation and nearly constant interest rates.

The direct implication is the emergence of a puzzle, which leaves a gap in the literature recent research is trying to close. Some possible explanations and competing exists, including the fiscal theory of the price level, advocated by Cochrane (2016). By concisely exposing the problem, we hope to facilitate the discussion of the possible solutions in future studies.

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References


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